

## IMPROVED SIMULATED ANNEALING FOR OPTIMIZATION OF VEHICLE ROUTING PROBLEM WITH TIME WINDOWS (VRPTW)

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### Abstrak

Vehicle routing problem with time windows (VRPTW) merupakan permasalahan optimasi kombinatorial yang banyak ditemui pada sistem distribusi permasalahan ini berkaitan dengan pengalokasian sejumlah kendaraan umum untuk melayani sejumlah konsumen, sejumlah konsumen mempunyai rentang waktu kesediaan yang berbeda dan harus dilayani dalam waktu tersebut. Paper ini memaparkan penggunaan metode simulated annealing yang diperkaya dengan beberapa fungsi khusus untuk menghasilkan solusi tetangga yang digunakan pada penelusuran are pencarian solusi dari VRPTW. Serangkaian percobaan menunjukkan bahwa simulated annealing yang diperkaya dengan fungsi-fungsi khusus dapat menghasilkan solusi yang baik dalam waktu rata-rata 82.29 detik.

Kata kunci: *Vehicle Routing Problem with Time Windows (VRPTW)*, Permasalahan optimasi kombinatoria, *Simulated annealing*, solusi tetangga.

### Abstract

*The Vehicle Routing Problem with Time Windows (VRPTW) is a combinatorial optimization problem that exists in various distribution systems. The problem deals with allocation of vehicles to service several customers, each customer has different available time, and the vehicles must visit the customers in their available time. This paper addresses the VRPTW by using an improved simulated annealing algorithm. Special functions to effectively exploring neighborhood solutions are developed. The functions are required to deal with the large search space of the VRPTW and enhance the power of the simulated annealing to obtain better solutions. The proposed approach is evaluated in comparison with well-known benchmark problems available in the literature. A set of computational experiments prove that the improved simulated annealing could produce promising results in the average of computational time of 82.29 seconds.*

*Keywords: Vehicle Routing Problem with Time Windows (VRPTW), combinatorial optimization problem, simulated annealing, neighborhood solution.*

## INTRODUCTION

The Vehicle Routing Problem (VRP) is a combinatorial optimization problem that exists in various distribution systems. The problem deals with allocation of vehicles to service several customers. One of the variances of VRP is Vehicle Routing Problem with Time Windows (VRPTW). The VRPTW has an additional constraint where each customer has different available time and the vehicles must visit the customers in their available time. The constraint is called *time window*.

The VRPTW is a complex problem and has many applications in real-life situations that require to minimize distribution costs [1][2]. Solving VRPTW problem can be considered as a part of decision making process in supply chain management [3]. An example application of the VRPTW is distribution of perishable foods [4]. In this case, the foods are assumed to decrease in value, throughout their lifetime. Thus, an efficient strategy is required to deliver the foods as quickly as possible. The quick distribution can be achieved by minimizing distribution routes. By minimizing distribution routes, a lower distribution cost is also obtained. A similar study can be found in [5] that focus in distribution of fresh vegetables.

As many combinatorial problems, the VRPTW belongs to NP-hard problem and time required to solve it will exponentially grow for larger problems [6]. Exact optimization methods and complete enumeration methods such as branch-and-bound may be difficult to solve these kinds of problems in reasonable amount of time [7]. Thus, a number of approaches have been proposed. Most of them are heuristic-based methods that have capability to solve complex combinatorial problems in reasonable amount of time [8][9].

This paper addresses the VRPTW by using improved simulated annealing algorithm. Special functions to effectively exploring neighborhood solution are developed. The functions are required to deal with a large search space of the VRPTW. The proposed approach is evaluated in comparison with well-known benchmark problems available in the literature. A set of computational experiments are carried out to prove the performance of the proposed approach.

Rest of the paper is organized as follow: First, the definition and assumptions of the VRPTW are detailed, followed by the discussion of related works on heuristic algorithms implementations for the VRPTW. The next section presents the development of specialized simulated annealing algorithm for the VRPTW, followed by the discussion of the results of computational experiments. Finally, the conclusion and future works are presented in the last section.

## VEHICLE ROUTING PROBLEM WITH TIME WINDOWS

The VRPTW can be defined as follow: there are a number of customers  $C$ , several vehicles  $V$ , and a depot  $d$ . Each customer has units of demand and the vehicles have maximum units to be carried. The aim of the problem is finding a set of routes starting and ending at  $d$ . Each customer in  $C$  must be visited by exactly one vehicle in  $V$ . An amount of time is required by the vehicle on the road and during servicing the customer.

An additional constraint called time window forces the vehicle to service the customers in their available time. The vehicle must wait if it arrives earlier than the opening time of the customer. However, late arrival of the vehicle is not permitted. Various objective functions have been addressed in the existing studies. However, most of the studies consider reducing the number of vehicles and the total travel distance of the vehicles simultaneously [10][11].

## RELATED WORKS

The complexity level of the VRPTW and its wide applicability to real-life situations have become reasons to do intensive research in this area. Various heuristic and exact optimization approaches have been proposed. For instance, Nazif and Lee [10] proposed a genetic algorithm to solve the VRPTW. The crossover operator of the genetic algorithm is optimized to satisfy constrains of the problem and also minimize total cost. Solomon's benchmark problems taken from [12] was used to evaluate the approach. They reported that their approach was very competitive in term of the quality of solutions.

Genetic algorithm was also used in a study by Wenjun [6]. The study improved classical genetic algorithms by implementing a greed randomized adaptive search method (GRASP) to generation an initial population. The study reported that the strategy was effective for obtaining better solutions of Solomon's benchmark problems

To produce better solutions, Yong and Yanfang [11] combined several methods to perform an approach called hybrid evolutionary algorithm. The hybrid method makes use the advantage of several methods including genetic algorithm, greedy randomized adaptive search procedure, the expanding neighborhood search strategy, and particle swarm optimization. This study also used Solomon's benchmark problems [12].

A hybrid approach was also developed by Baños, et al. [8] that combined evolutionary algorithm and simulated annealing. Solomon's benchmark problems were also used in this study. The approach was further improved to address the VRPTW with multiple objectives. Besides minimizing total traveling distances, the approach tried to balance the distance traveled by each vehicle.

Even though simulated annealing is a simple method, it can be used to solve a complex problem such as the VRPTW if it is implemented properly. For example, Baños, et al. [13] developed a parallel simulated annealing to explore the search space of the VRPTW effectively. They claimed that parallel simulated annealing gave good results on Solomon's benchmark problems.

Simulated annealing was also used in variant of the VRPT in work by Afifi, et al. [14]. In this case, a customer may require simultaneous visits from different vehicles. The simulated annealing was equipped with several local searches that were claimed producing good solutions comparable to those achieved by other approaches in the literature. A similar variant of the VRPTW was also solved using simulated annealing in a study by Lin, et al. [9]. As there were no benchmark instances for their VRPTW, the Solomon's benchmark problems were modified in their computational experiments.

An easy implementation and good result reported are becoming reason for the author to use simulated annealing to solve the VRPTW. A special mechanism to produce good

neighborhood solutions is implemented to enhance the power of the proposed simulated annealing. Rather than using only a single operator to produce neighborhood solutions, the mechanism applies four operators that are specially designed to explore the search space of the VRPTW.

## DEVELOPMENT OF SIMULATED ANNEALING FOR THE VRPTW

As a generic probabilistic method, simulated annealing has capability to obtain near optimum solution for complex problems with large search space. The method has been successfully implemented for various combinatorial problems [9][15].

Simulated annealing method is inspired by the physical process of annealing where a material such as steel or glass is heated and then cooled. By simulate the physical process; each step of the simulated annealing algorithm replaces the current solution by neighbor solution. The neighbor solution is randomly generated using functions/operators that are developed specifically according to the problem [16].

If the new (neighbor) solution improves the objective function, then it accepts the exchange and the new solution are preserved. Otherwise, the new solution is accepted with a probability that depends both on the difference between the objective function values and also on a global parameter  $T$  (called the temperature), that is gradually decreased during the process. This mechanism make simulated annealing almost changes the current solution randomly when  $T$  is large in earlier iterations. However, the current solution is gradually to be driven to the optimum area as  $T$  is being decreased.

The allowance for accepting a worse solution enables simulated annealing avoiding local optimum areas [16]. This feature make simulated annealing differs with simple hill-climbing search method that accepts only a better solution in each iteration [17].

Simulated annealing requires three main processes to solve the VRPTW as follows:

1. Generating a random initial solution.
2. Producing a new solution by using special neighbor operators.
3. Applying a function to reduce the probability of accepting a new worse solution which is useful in escaping local

optimums during the course of the search for the global optimum.

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BEGIN Simulated Annealing
Let  $t = temp0$ 
Generate random initial solution  $S$  as
current solution
WHILE  $t \geq temp1$ 
  REPEAT  $inner\_itr$  times
    Let  $d1$  = value of the objective
      function (distance) of  $S$ 
    Produce  $S_n$  (neighbor of  $S$ ) as a
      new solution
    Let  $d2$  = value of the objective
      function (distance) of  $S_n$ 
    IF  $d2 < d1$  THEN
accept change ( $S \leftarrow S_n$ )
      Let  $S_b \leftarrow S_{new}$ 
    ELSE IF  $d2 > d1$  THEN
accept change with probability
       $prob = \frac{1}{e^{\frac{(d2-d1)k}{d1t}}}$ 
    END IF
  END REPEAT
  Let  $t \leftarrow t * cooling\_factor$ 
END WHILE
Return  $S_b$  as the best solution
END Simulated Annealing

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**Figure 1.** Pseudo-code of Simulated Annealing

A pseudo code of simulated annealing that consist the three main processes is presented in Fig 1. The temperature  $t$  is started with value of  $temp0$ .  $inner\_itr$  is a variable that is used to determine number of iterations for exploiting a local search area. This property enable the simulated annealing to obtain the best solution in the local area and to balance its power for exploitation and exploration in the search area.

The cycle of the simulated annealing will be terminated after  $t$  reaches final temperature  $temp1$ . A variable named  $cooling\_factor$  is used to gradually decrease the value of  $t$ .

As other heuristic methods, setting proper values for the parameters of algorithms is critical to achieve good result in reasonable amount of time [18][19]. The best value of a given parameter is obtained by changing the values of that parameter in certain range while keeping all other parameters constant [20]. After a set of preliminary experiments, the

parameters for the proposed simulated annealing heuristic are set as follows:

1. the initial temperature  $temp0$  is 0.8,
2. the  $cooling\_factor$  is 0.995,
3. the final temperature  $temp1$  is 0.01,
4. the number of iterations per fixed temperature ( $inner\_iteration$ ) is set to 6000,

A value of  $k$  is used to calculate the probability of accepting a worse new solution. Thus, determining a proper value  $k$  is critical. Setting this value too low will cause a random change of the current solution as the probability of accepting the worse new solution is too high. In contrast, a high value of  $k$  will produce a low probability of accepting the worse new solution which means losing the ability to escape local optimums. After several experiments, the proper value of  $k$  is set to 25.

Instead of generating a random initial solution, the initial solution  $S$  is produced by using simple insertion method proposed by Solomon [12]. The method requires a parameter called  $\alpha$ . In this paper, the value of  $\alpha$  is set to 0.5. Note that infeasible solution that violates time window constraints can be resulted on certain problems. However, after a few iterations, the simulated annealing can repair the solution becoming feasible.

The simulated annealing is designed to fully explore and exploit the wide search space of the VRPTW. Thus, instead of using a single operator, a new neighbor solution is randomly generated using operators that are specially developed for the VRPTW. Four operators have been developed for this purpose as follows:

1. *Move1*: internal moving within a route of a vehicle. It is carried out by changing position/sequence of a customer to reduce the total distance of the route.
2. *Move2*: external moving between routes. The operator moves a customer to other vehicle to enable the reduction of the total distance. At the beginning step of the simulated annealing, this operator tends to increase the number of used vehicles.
3. *Move3*: external moving from a vehicle with least customer. This operator will effectively reduce the number of used vehicles. Thus, the total distance is also

reduced as each vehicle must travel from and returns back to the starting point.

4. *Move4*: external moving from a vehicle with longest distance. This operator is used to speed up the simulated annealing obtaining a shorter total distance.

On each iteration, the simulated annealing randomly chooses which operator should be used with probability of 0.3, 0.3, 0.3, and 0.1 for *Move1*, *Move2*, *Move3*, and *Move4* respectively. These values are determined by using several preliminary experiments. Note that the simultaneous application of the operators will enable the simulated annealing to properly determining the number of vehicles that minimizes the total distance.

It is possible that the operator produce infeasible solutions that violate the constraints of the VRPTW. For example, some customers cannot be visited in their time window or a vehicle cannot supply demands of the customer in its route. Thus, a penalty value is applied to prevent the simulated annealing accepting the infeasible solutions.

## RESULT AND DISCUSSION

The performance of the proposed simulated annealing is evaluated using a classical set of 28 benchmark problems developed by Solomon [12]. The proposed simulated annealing is implemented in Java and experiment is carried out on personal computer equipped with AMD Quad-Core processor working at speed 2.8 GHz and 4GB DDR3 memory. Simulated annealing is a stochastic method, so different solution is obtained in each run. To obtain a fair result, the proposed simulated annealing is run 5 times for each benchmark problem with different initial solutions.

The experimental results are presented in Table 1. The best result of the simulated annealing is compared to the best results produced by other heuristic approaches that are available in the literature. The data can be found in “<http://w.cba.neu.edu/~msolomon/problems.htm>”.

Three sets of problems are used. The first set (R101 to R112) contains randomly generated customer coordinates. The other sets

(RC101 to RC108, RC201 to RC208) contain mix of random and clustered customers. Each problem has 100 customers as indicated in the last 3 digits of the problem name.

Column 'NV' of Table 1 shows the number of used vehicles whereas column 'Distance' shows the total distance traveled by all vehicles. The computational time (in seconds) required by the improved simulated annealing is also presented in column 'Time'.

To measure the quality of solutions produced by the proposed simulated annealing, a relative deviation (*dev*) of the total distance obtained by the proposed simulated annealing to the best solution obtained by the other heuristic method is calculated using Equation (1) as follow:

$$dev = \frac{SA\ result - Best\ result}{SA\ result} 100\% \quad (1)$$

By using only simple neighborhood operators, the proposed simulated annealing produces promising results with the average of deviations is only 3.56%. The results are achieved with the average of computational time of 82.29 seconds. Thus, it proves the effectiveness of the neighborhood operators to deal with the complexity of the VRPTW. Furthermore, the proposed simulated annealing could produce better results in some problems (R103.100, RC101.100, RC102.100, RC105.100, RC106.100, RC202.100) comparable to those achieved by other heuristic methods.

Only in few problem (RC203.100, RC204.100, RC206.100, RC208.100) the proposed simulated annealing produce deviations more than 10%. It should be understood that the neighborhood operators are not designed to handle clustered route.

While the simulated annealing considers only minimizing the total distance, better results are highly correlated to the fewer number of vehicles as shown in Table 1. The results prove that operator *Move3* that is used to reduce the number of vehicles is effective.

Table 1. The Experimental Results

Problem	Previous Heuristic Result			Improved Simulated Annealing			
	NV	Distance	Authors	NV	Distance	Time	Dev
R101.100	19	1645.79	H	19	1698.81	99	3.22
R102.100	17	1486.12	RT	18	1515.27	79	1.96
R103.100	13	1292.68	LLH	15	1259.173	102	-2.59
R104.100	9	1007.24	M	12	1029.822	74	2.24
R105.100	14	1377.11	RT	16	1444.944	100	4.93
R106.100	12	1251.98	M	13	1277.005	95	2.00
R107.100	10	1104.66	S97	12	1139.83	94	3.18
R108.100	9	960.88	BBB	11	997.1493	94	3.77
R109.100	11	1194.73	HG	12	1199.282	95	0.38
R110.100	10	1118.59	M	12	1132.535	75	1.25
R111.100	10	1096.72	RGP	12	1104.495	96	0.71
R112.100	9	982.14	GTA	10	1029.36	74	4.81
RC101.100	14	1696.94	TBGGP	15	1674.301	97	-1.33
RC102.100	12	1554.75	TBGGP	15	1505.954	74	-3.14
RC103.100	11	1261.67	S98	11	1330.183	73	5.43
RC104.100	10	1135.48	CLM	10	1160.971	72	2.24
RC105.100	13	1629.44	BBB	16	1587.483	79	-2.57
RC106.100	11	1424.73	BBB	12	1395.607	75	-2.04
RC107.100	11	1230.48	S97	12	1280.14	76	4.04
RC108.100	10	1139.82	TBGGP	11	1195.784	76	4.91
RC201.100	4	1406.91	M	5	1502.577	73	6.80
RC202.100	3	1367.09	CC	4	1294.543	75	-5.31
RC203.100	3	1049.62	CC	4	1203.397	72	14.65
RC204.100	3	798.41	M	4	892.8673	70	11.83
RC205.100	4	1297.19	M	4	1413.685	73	8.98
RC206.100	3	1146.32	H	4	1274.372	75	11.17
RC207.100	3	1061.14	BVH	4	1103.664	70	4.01
RC208.100	3	828.14	IKMUY	3	945.5482	97	14.18
				<b>average</b>	<b>82.29</b>	<b>3.56</b>	

## CONCLUSION

This paper addresses the VRPTW by using improved simulated annealing algorithm. Special functions to effectively exploring neighborhood solutions are developed. The functions are required to deal with a large search space of the VRPTW. The proposed approach is evaluated in comparison with well-known benchmark problems available in the literature. A set of computational experiments prove that the improved simulated annealing

could produce promising results in the average of computational time of 82.29 seconds.

One drawback of the neighborhood operators are they cannot effectively deal with clustered data. Thus, the next research will consider this problem by combining the simulated annealing with other methods. For example, clustering the data using a method such as *k-mean clustering* is done in the first stage and the simulated annealing is applied in the second stage.

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